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Distributed Beamforming in Wireless Relay Networks with Quantized Feedback

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Abstract

This paper is on quantized beamforming in wireless amplify-and-forward (AF) relay networks. We use the Generalized Lloyd Algorithm (GLA) to design the quantizer of the feedback information and specifically to optimize the bit error rate (BER) performance of the system. Achievable bounds for different performance measures are derived. First, we analytically show that a simple feedback scheme based on relay selection can achieve full diversity. Unlike the previous diversity analysis on the relay selection scheme, our analysis is not aided by any approximations or modified forwarding schemes. Then, for high-rate feedback, we find an upper bound on the average signal-to-noise ratio (SNR) loss and show that it decays at least exponentially with the number of feedback bits, B. Using this result, we also demonstrate that the capacity loss also decays at least exponentially with B. In addition, we provide approximate upper and lower bounds on the BER, which can be calculated numerically. Simulations are also provided, which confirm our analytical results. We observe that, for R relays, our designs achieve full diversity when $B \geq \log R$ and a few extra feedback bits are sufficient for a satisfactory performance in terms of the array gain. Simulations also show that our approximate BER is a reliable estimation on the actual BER for even moderate values of B.

Index Terms

Wireless relay networks, beamforming, vector quantization, bit error probability, diversity order.

I. INTRODUCTION

Multiple antennas can improve the performance of communication systems by introducing diversity. The diversity benefit is a result of having statistically independent communication

paths between the transmitter and the receiver. However, the use of multiple antennas may not be practical due to the power and cost limitations of the mobile devices. Recent developments showed that such difficulties can be overcome by the concept of cooperation, in which several relays assist the communication between the terminals of the network. In this case, each relay provides an independent communication path, and this results in a cooperative diversity gain [1]–[3].

Many cooperative schemes have been proposed [4]–[7], among which non-regenerative schemes such as amplify-and-forward (AF) have especially become popular due to their low complexity. When channel state information (CSI) is not available at the relay terminals, distributed space-time coding schemes have been established [5], [8], [9], which achieve full diversity. On the other hand, with available CSI, each relay should adjust its transmit power and its transmit signal phase for a better performance. This adaptive network design concept is analogous to beamforming in multiple antenna systems, and is thus called network beamforming [10].

The beamforming problem for networks with parallel relays is studied in [11]–[14] for a sum-power constraint on relays, and in [10], [12], for networks with individual power constraints. Both works require that each relay knows its own channels perfectly, in addition to one or two real numbers that can be calculated by the receiver and common for all relay nodes. A more practical assumption is that there is only partial channel information at the relay nodes.

There are two widely used types of partial channel information: channel statistics (means and covariances) and quantized instantaneous CSI. The first case was addressed in [15]. In this work, we focus on the latter. In [16], outage minimization of single-relay networks with quantized CSI is performed. It is assumed that there is a long-term power constraint on the total power of the transmitter and the relay. A numerical treatment for general parallel AF networks with quantized CSI can be found in [17]. Note that, a special case of quantized feedback is relay selection [18]–[24], which uses a fixed number of $\log R$ feedback bits for a network with R relays.

Although the concept of network beamforming with quantized channel information is highly similar to quantized beamforming in multiple-antenna systems [25]–[29], we cannot directly apply the results on the latter. This is due to the existence of the extra channels between the transmitter and the relays that results in noise amplification at the relay terminals.

In this work, we analyze networks with any number of relays, in which each relay has its own short-term power constraint. We assume full channel information at the receiver, but each relay only knows its own receiving channel and a partial CSI provided by B bits of feedback. The feedback bits are conveyed from the receiver to the relays and they represent a quantized network beamforming vector. This beamforming vector is selected from a codebook which is known at the relays and the receiver. Our performance measure is the bit error rate (BER).

The paper is organized as follows: In Section II, we introduce our network model and the problem definition. In Section III, we describe our codebook design strategy based on the Generalized Lloyd Algorithm (GLA). In Section IV, we investigate the achievable diversity of the optimal relay selection scheme, and show that B feedback bits can achieve diversity 2^B for $B = 0, \log 2, \ldots, \log R$. In Section V, we provide an upper bound on the average signal-to-noise ratio (SNR) loss for high-rate feedback and also extend this result for the capacity loss. In Section VI, we derive approximate upper and lower bounds on the BER, which give an accurate description of the actual BER for moderate and large values of B. The numerical results are provided in Section VII. Finally, in Section VIII, we draw our major conclusions. Some technical proofs are provided in the appendices.

Notation: $|| \cdot ||$ indicates the 2-norm, $\langle \cdot | \cdot \rangle$ is the inner product. $\mathbb{Z}^+$, $\mathbb{R}$, and $\mathbb{C}$ represent the sets of positive integers, real numbers and complex numbers, respectively. For $z \in \mathbb{C}$, $|z|$ indicates the absolute value, z^* denotes the complex conjugate, and $\mathfrak{Re}(z)$ is the real part of z. $\arg z$ is the complex argument of z with $0 \leq \arg z < 2\pi$. $\mathrm{E}[\cdot]$ is the expected value and Pr represents the probability. $f_X(\cdot)$ is the probability density function (PDF) of the random variable X. $X \sim \mathrm{Exp}(\zeta_x)$ means that X is an exponential random variable distributed as $f_X(x) = \zeta_x \exp(-\zeta_x x)$ for $x \geq 0$ and $f_X(x) = 0$ for $x < 0$. For any sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} - \mathcal{B}$ is the set of elements in $\mathcal{A}$, but not in $\mathcal{B}$. $A \subset B$ means $\mathcal{A}$ is a subset of $\mathcal{B}$, and $\mathcal{A} \cap \mathcal{B}$ is the intersection of $\mathcal{A}$ and $\mathcal{B}$. $|\mathcal{A}|$ is the cardinality of $\mathcal{A}$ and $\mathcal{A}^r = \{(a_1, \ldots, a_r) | a_1, \ldots, a_r \in \mathcal{A}\}$, $r \in \mathbb{Z}^+$, is the cartesian power. Finally, $\emptyset$ is the empty set, $\mathrm{Q}(\cdot)$ represents the Gaussian tail function, $\Gamma(\cdot)$ is the gamma function and $K_\nu(\cdot)$ is the modified Bessel function of the second kind of order ν.

II. NETWORK MODEL AND PROBLEM STATEMENT

A. System Model

The block diagram of the system is shown in Fig. 1. We have a relay network with one transmitter-receiver pair and R parallel relays. We assume that there is no direct link between the transmitter and the receiver. Denote the channel from the transmitter to the ith relay by f_i and the channel from the ith relay to the receiver by g_i. Let $\mathbf{h} = (f_1, \ldots, f_R, g_1, \ldots, g_R)$ be the corresponding channel vector. The entries of $\mathbf{h}$ are assumed to be independent, and each entry is modeled as $\mathcal{CN}(0, 1)$, a complex Gaussian random variable with variance $1/2$ per complex dimension.

Only the short-term power constraint is considered, which means that there is an upper bound on the power used at each node, and for every symbol transmission, the power levels used at the transmitter and the ith relay are no larger than P_0 and P_i, respectively.

We assume a quasi-static channel model, in which the channel realizations vary independently from one frame to another, while within each frame the channels remain constant. We assume that the receiver knows $\mathbf{h}$ and each relay has perfect knowledge of its own receiving channel, namely the ith relay knows f_i. Each relay also has B bits of partial CSI, provided by the receiver feedback. The feedback channels are assumed to be error-free and delay-free. Note that within each frame, the information each relay receives from the feedback is the same. With this property, there is a common codebook $\mathcal{C} = \{\mathbf{x}^1, \ldots, \mathbf{x}^M\}$ of cardinality $M = 2^B$, which is designed offline and known by all the relay terminals and the receiver.

The feedback transmission scheme operates as follows: For each frame, the channel realization $\mathbf{h}$ is quantized using a quantizer $\mathcal{Q}$ defined by the encoder and the decoder mappings $\mathcal{E} : \mathbb{C}^{2R} \to \mathcal{I}$ and $\mathcal{D} : \mathcal{I} \to \mathcal{C}$, where $\mathcal{I} \triangleq \{1, \ldots, M\}$ represents the index set for the codebook elements. The encoding operation is performed at the receiver and the feedback bits represent the encoder output. Then, each relay decodes the feedback bits using $\mathcal{D}$, to find the corresponding codebook element. If $\mathcal{E}(\mathbf{h}) = m$, we have $\mathcal{D}(m) = \mathbf{x}^m \in \mathcal{C}$. For the rest of this paper, we will use the well-known representation $\mathcal{Q}(\mathbf{h}) \triangleq \mathcal{D}(\mathcal{E}(\mathbf{h}))$ as the combined effect of the mappings $\mathcal{E}$ and $\mathcal{D}$. Therefore, $\mathcal{Q} : \mathbb{C}^{2R} \to \mathcal{C}$ and $\mathcal{Q}(\mathbf{h}) = \mathbf{x}^m$, for some $m \in \mathcal{I}$ and $\mathbf{x}^m \in \mathcal{C}$.

B. Transmission Scheme and Performance Measure

We consider a two-step AF protocol [10]. During the first step, the transmitter selects a symbol s from a constellation $\mathcal{S}$, and sends $\sqrt{P_0}s$. We normalize s as $\mathrm{E}[|s|^2] = 1$. Therefore, the average power used at the transmitter is P_0. During this step, there is no reception at the receiver, but the ith relay receives $r_i = \sqrt{P_0}f_i s + \nu_i$. The noises ν_i are independent and modeled as $\mathcal{CN}(0,1)$.

One of the key issues in cooperative network design is the relay operation. In this work, we use AF relays. However, unlike [4]–[6], we incorporate the idea of network beamforming [10], by allowing each relay to adaptively adjust its transmit power and phase according to the feedback information. Thus, each element of the codebook $\mathcal{C}$ corresponds to a beamforming vector.

Assume that the receiver feeds back the index m. Then, the relays use the beamforming vector $\mathbf{x}^m = (x_1^m, \dots, x_R^m)$. In the second step, the ith relay sends $t_i = x_i^m \sqrt{\rho_i} r_i$ where $\rho_i \triangleq \frac{P_i}{1+|f_i|^2 P_0}$. The average transmit power of the ith relay can be calculated as $|x_i^m|^2 P_i$, where $0 \leq |x_i^m| \leq 1$ is a result of the short term power constraint.

After these two steps of transmission, the received signal at the receiver can be expressed as:

$$y = \sqrt{P_0} \left(\sum_{i=1}^{R} x_i^m f_i g_i \sqrt{\rho_i} \right) s + \sum_{i=1}^{R} x_i^m g_i \sqrt{\rho_i} \nu_i + \omega,$$

where ω is the noise at the receiver, which is also distributed as $\mathcal{CN}(0,1)$ and independent from ν_i. The received SNR can be given as:

$$\gamma(\mathbf{h}; \mathcal{Q}, \mathcal{C}) = P_0 \frac{\left| \sum_{i=1}^{R} x_i^m f_i g_i \sqrt{\rho_i} \right|^2}{1 + \sum_{i=1}^{R} |x_i^m|^2 |g_i|^2 \rho_i} \tag{1}$$

for channel state $\mathbf{h}$ with the quantizer $\mathcal{Q}$ and the beamforming codebook $\mathcal{C}$. We will also frequently use the notation $\gamma(\mathbf{x}, \mathbf{h})$ to describe the received SNR for a specific beamforming vector $\mathbf{x}$. Note that given $\mathcal{C}$, $\gamma(\mathbf{h}; \mathcal{Q}, \mathcal{C}) = \gamma(\mathbf{x}, \mathbf{h})$ whenever $\mathbf{x} = \mathcal{Q}(\mathbf{h})$.

In this paper, our performance measure is the BER, which can be simply expressed as an expectation over the channel state vector:

$$\Pr(E|\mathcal{Q}, \mathcal{C}) = \mathrm{E}_{\mathbf{h}} \left[\mathrm{Q} \left[\sqrt{2\beta \gamma(\mathbf{h}; \mathcal{Q}, \mathcal{C})} \right] \right], \tag{2}$$

where β is a constant depending on the constellation S.

C. Problem Statement

Our network beamforming design problem is the joint optimization of the quantizer and the quantization codebook, given a fixed number of feedback bits B, such that the BER is minimized.

The quantizer design is straightforward:

Lemma 1: For any beamforming codebook $\mathcal{C} = \left\{ \mathbf{x}^1, \ldots, \mathbf{x}^M \right\}$, the optimal quantizer $\mathcal{Q}^*$ to minimize the BER is given by:

$$\mathcal{Q}^*(\mathbf{h}) = \arg \max_{\mathbf{x} \in \mathcal{C}} \gamma(\mathbf{x}, \mathbf{h}). \tag{3}$$

Proof: Consider a fixed codebook $\mathcal{C}$ and an arbitrary $\mathcal{Q}' : \mathbb{C}^{2R} \to \mathcal{C}$. For any $\mathbf{h}$, we have $\gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}) \geq \gamma(\mathbf{h}; \mathcal{Q}', \mathcal{C})$ and the result follows from the monotonicity of the Q function. ■

In other words, given $\mathcal{C}$, the optimal quantizer chooses the SNR maximizing beamforming vector in $\mathcal{C}$. From now on, we fix the quantizer operation to be $\mathcal{Q}^*$. Therefore, the codebook uniquely determines the performance of our feedback system. Before introducing the codebook design methods, we give a necessary condition for a codebook to be optimal.

When there is no limit on the number of feedback bits, the relays can always use the optimal beamforming vector, which was derived in [10]. In this case, regardless of the channel state, at least one relay should transmit with full power. This requirement can be represented by the set:

$$\mathcal{X} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } ||\mathbf{x}||_\infty = 1\}, \tag{4}$$

where $||\mathbf{x}||_\infty \triangleq \max_i |x_i|$. For our network model with finite feedback bits, we show that each vector in an optimal codebook should also be an element of the set $\mathcal{X}$:

Lemma 2: If $\mathcal{C}^*$ is optimal, then $\mathcal{C}^* \subset \mathcal{X}$.

Proof: Suppose an optimal codebook $\mathcal{C}'$ with $\mathcal{C}' - \mathcal{X} \neq \emptyset$ exists. So, there exists $\mathbf{x}^k \in \mathcal{C}'$ where $|x_i^k| < 1$, $\forall i \in \{1, \ldots, R\}$. Define $\eta \triangleq \max_i |x_i^k|$ and let $\mathbf{y} = \frac{1}{\eta} \mathbf{x}^k$. Replace $\mathbf{x}^k$ in $\mathcal{C}'$ with $\mathbf{y}$, which results in a new codebook $\mathcal{C} = \{\mathbf{x}^1, \ldots, \mathbf{x}^{k-1}, \mathbf{y}, \mathbf{x}^{k+1}, \ldots, \mathbf{x}^M\}$. From (1), it is easy to see that $\gamma(\mathbf{y}, \mathbf{h}) > \gamma(\mathbf{x}^k, \mathbf{h})$, $\forall \mathbf{h} \in \mathbb{C}^{2R}$. Therefore, $\mathrm{P}(E|\mathcal{Q}^*, \mathcal{C}) < \mathrm{P}(E|\mathcal{Q}^*, \mathcal{C}')$ and $\mathcal{C}'$ is suboptimal. This contradicts the optimality of $\mathcal{C}'$. ■

This lemma simply states that at least one element of every beamforming vector in an optimal codebook should have unit norm. Therefore, the set $\mathcal{X}$ completely defines the feasible set for the

codebook vectors. Note that, for networks with a sum-power constraint on relays, the feasible set of codebook vectors can similarly be defined by the set $\{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } ||\mathbf{x}||^2 = 1\}$.

III. CODEBOOK DESIGN: THE GENERALIZED LLOYD ALGORITHM

We use the Generalized Lloyd Algorithm (GLA) to design optimal codebooks of an arbitrary size. The algorithm is based on two optimality conditions, which can be re-stated as follows:

For a given codebook $\{\mathbf{x}^m : m = 1, \ldots, M\}$, the optimal partition of the channel state space is given by:

$$\mathcal{R}_m \triangleq \{\mathbf{h} : \gamma(\mathbf{x}^m, \mathbf{h}) > \gamma(\mathbf{x}^n, \mathbf{h}), \forall n \neq m\}, \tag{5}$$

and for a given partition $\{\mathcal{R}_m : m = 1, \ldots, M\}$, the optimal codebook vectors satisfy:

$$\mathbf{x}^m = \arg \min_{\mathbf{x} \in \mathcal{X}} \mathrm{E}_\mathbf{h} \left[\mathrm{Q} \left(\sqrt{2\beta\gamma(\mathbf{x}, \mathbf{h})} \right) | \mathbf{h} \in \mathcal{R}_m \right]. \tag{6}$$

In general, the algorithm is initialized with a random choice of codebook vectors. Then, the optimality conditions are iterated until convergence is achieved. Note that the GLA does not guarantee global optimality. However, at each iteration, the algorithm provides an improved codebook design and each new codebook results in a lower BER. Therefore, it is guaranteed that the algorithm converges and the resulting codebook achieves at least the BER performance of the initialization codebook.

The GLA can also be used for designing beamforming codebooks for networks with a sum-power constraint on the relays. The only difference is in the shape of the feasible set. Moreover, all the diversity and scaling results we will obtain for networks with individual power constraints apply to networks with a sum-power constraint as well.

IV. DIVERSITY ANALYSIS OF RELAY SELECTION SCHEME

The maximal spatial diversity of networks with R relays is R. In this section, we will prove that a simple feedback scheme based on relay selection achieves this full diversity. Note that we can also improve the relay selection codebook by GLA. This way, we make sure that the designed codebook provides full diversity and also better performance compared to relay selection.

For the relay selection scheme, only one of the R relays is allowed to cooperate for a given

constant fading block. The corresponding codebook is given by $\mathcal{C}_\mathrm{e} \triangleq \{\mathbf{e}^m : m = 1, \ldots, M\}$, where $\mathbf{e}_i^m = 1$ if $i = m$, and $\mathbf{e}_i^m = 0$ for $i \neq m$. Using (1) and (3), the received SNR is

$$\gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}_\mathrm{e}) = \max_{i \in \{1, \ldots, R\}} \frac{P_0 P_i \, |f_i g_i|^2}{1 + |f_i|^2 \, P_0 + |g_i|^2 \, P_i}. \tag{7}$$

A BER analysis for this scheme was previously performed in [20]–[22], which however, use approximations on (7). In this paper, we find a rigorous upper bound on the BER.

Theorem 1: Let $P_i = \lambda_i P_0$, where $\lambda_i > 0$ are constants. Define $\lambda \triangleq \max_i \{1 + \lambda_i^{-1/2}\}$, which is also a constant. Then, the BER of the relay selection scheme is bounded by:

$$\Pr(E | \mathcal{Q}^*, \mathcal{C}_\mathrm{e}) < R P_0^{-R} \lambda^{2R} \sum_{n=0}^{R-1} \binom{R-1}{n} \beta^{-(n+1)} n!. \tag{8}$$

Proof: See Appendix I. ∎

Notice that the exponent of P_0 in (8) is $-R$. Thus, the relay selection scheme achieves the full diversity of R with $M = R$. Similarly, diversity order M is achievable for $M < R$, simply by considering the selection scheme for any fixed M of the R relays and disregarding the others.

As we have discussed before, we can design a new quantization codebook to achieve better array gain compared to the relay selection codebook using the GLA. Since the new codebook also provides the same full diversity order property as the relay selection scheme, its superior average SNR results in a better array gain.

V. AVERAGE SNR LOSS WITH QUANTIZED FEEDBACK

In this section, the high-rate feedback scenario is considered. We introduce a systematic codebook design $\mathcal{C}_u$, which also incorporates the relay selection vectors. Thus, the resulting design provides full diversity order. In addition, we show that the average SNR loss of the design decays exponentially with the number of feedback bits, which indicates an increasing array gain. We also use this result to find an upper bound on the capacity loss.

A. Codebook Design

A beamforming vector contains both power and phase information. For the design of $\mathcal{C}_u$, we treat them separately and construct two different codebooks to represent each. Then, the two codebooks are combined to construct $\mathcal{C}_u$.

Quantized phase codebook: We write the received SNR as $\gamma(\mathbf{x}, \mathbf{h}) = P_0 \frac{\left| \sum_{i=1}^{R} |x_i| f_i g_i \sqrt{\rho_i} e^{j\Delta_i} \right|^2}{1 + \sum_{i=1}^{R} |x_i|^2 |g_i|^2 \rho_i}$,

where $\Delta_1 = 0$ and $\Delta_i = \arg x_i - \arg x_1$, $i = 2, \ldots, R$. Without loss of generality, we only need to quantize $\Delta_2, \ldots, \Delta_R$. For each $\Delta_i \in [0, 2\pi)$, we use a scalar quantizer defined by the centroids $U \triangleq \left\{ 0, 2\pi \frac{1}{L}, \ldots, 2\pi \frac{L-1}{L} \right\}$, $L \geq 2$, $L \in \mathbb{Z}^+$. Then, we use the set of vectors $\mathcal{C}_{\text{phase}} \triangleq U^{R-1}$ to quantize the phase.

Quantized power codebook: Similarly, we define $V \triangleq \left\{ 0, \frac{1}{K}, \ldots, \frac{K-1}{K}, 1 \right\}$, $K \geq 2$, $K \in \mathbb{Z}^+$, and consider the set V^R. As a result of Lemma 2, we remove the vectors in V^R but not in $\mathcal{X}$, which results in $\mathcal{C}_{\text{power}} \triangleq V^R \cap \mathcal{X} = V^R - (V - \{1\})^R$. Fig. 2 depicts an example for the construction of the quantized power codebook.

Therefore, we have $|\mathcal{C}_u| = |\mathcal{C}_{\text{phase}}||\mathcal{C}_{\text{power}}| = L^{R-1} \left[K^R - (K-1)^R \right]$.

B. Average SNR Loss

The *pointwise* SNR loss of $\mathcal{C}_u$ can be defined as $D(\mathbf{h}) \triangleq \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{X}) - \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}_u)$, which is the difference between the received SNRs with perfect feedback and with $\mathcal{C}_u$.[1]

Theorem 2: The average SNR loss with quantized feedback decays at least linearly as the number of quantization vectors $M = 2^B$, or exponentially as the number of feedback bits B:

$$\mathrm{E}_{\mathbf{h}}\left[D(\mathbf{h}) \right] < \mu 2^{-\frac{B}{2(R-1)}}, \tag{9}$$

where μ is independent of B.

Proof: See Appendix II. ∎

Using Theorem 2, we can find a similar result for capacity loss with quantized feedback.

C. Capacity Loss

Let C and C_u denote the ergodic capacities with perfect feedback and with the codebook $\mathcal{C}_u$, respectively. Note that $C = \mathrm{E}_{\mathbf{h}}\left[\log(1 + \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{X})) \right]$ and $C_u = \mathrm{E}_{\mathbf{h}}\left[\log(1 + \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}_u)) \right]$, both in nats per channel use. Then, the capacity loss $C_L \triangleq C - C_u$ can be bounded as:

$$C_L = \mathrm{E}_{\mathbf{h}}\left[\log\left(1 + \frac{D(\mathbf{h})}{1 + \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}_u)} \right) \right] \leq \mathrm{E}_{\mathbf{h}}\left[\frac{D(\mathbf{h})}{1 + \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}_u)} \right] \leq \mathrm{E}_{\mathbf{h}}[D(\mathbf{h})] \leq \mu 2^{-\frac{B}{2(R-1)}}, \tag{10}$$

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[1]By $\mathcal{Q}^*$, we mean the optimal quantizer with a particular codebook, thus the two $\mathcal{Q}^*$s in the SNR loss are actually different. However, with an abuse of notation, we omit the dependence of $\mathcal{Q}^*$ to the codebook, as it is clear from the context.

where the first inequality is a result of the fact that $\log(1 + x) \leq x$ for $x \geq 0$.

We note that the constant μ will not result in a tight upper bound for moderate M, therefore (9) and (10) are useful especially with high rate feedback. However, these results show how the average SNR loss and the capacity loss scale with the number of feedback bits.

VI. BIT ERROR PROBABILITY WITH QUANTIZED FEEDBACK

In this section, we analyze the BER of the network. One big challenge for the BER analysis comes from the shape of the feasible set of the codebook vectors $\mathcal{X}$. Even in the case of perfect channel knowledge at the relay terminals, this leads to complicated expressions for the optimal beamforming vector, and consequently, great difficulties in the derivation of the system performance. To overcome these problems, we bound the BER with feasible set $\mathcal{X}$ by BERs with feasible sets that are much easier to deal with.

A. Performance Bounds Using Sum-Power Constraints

For any beamfoming vector $\mathbf{x}$, first, we consider the restriction $\|\mathbf{x}\|^2 = 1$, which just implies a sum-power constraint on relays. The corresponding feasible set for the codebook vectors is $\mathcal{X}^{\text{in}} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } \|\mathbf{x}\|^2 = 1\}$. Similary, we define $\mathcal{X}^{\text{out}} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } \|\mathbf{x}\|^2 = R\}$. Note that for any codebook $\mathcal{C}^{\text{in}} \subset \mathcal{X}^{\text{in}}$ (or $\mathcal{C}^{\text{out}} \subset \mathcal{X}^{\text{out}}$), the optimal encoder is also $\mathcal{Q}^*$.

Lemma 3: Let $\mathcal{C}^{\text{in}*} \subset \mathcal{X}^{\text{in}}$, $\mathcal{C}^* \subset \mathcal{X}$ and $\mathcal{C}^{\text{out}*} \subset \mathcal{X}^{\text{out}}$ be the optimal codebooks of cardinality M for their respective feasible sets. Then, $\Pr(E|\mathcal{Q}^*, \mathcal{C}^{\text{out}*}) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}^*) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}^{\text{in}*})$.

Proof: First, we prove $\Pr(E|\mathcal{Q}^*, \mathcal{C}^*) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}^{\text{in}*})$. For any $\mathbf{y}^m \in \mathcal{C}^{\text{in}*}$, choose $\eta_m = \max_i |y_i^m|$, and construct a new codebook $\mathcal{C} = \{\frac{1}{\eta_m}\mathbf{y}^m, m = 1, \ldots, M\}$. Note that $\mathcal{C} \in \mathcal{X}$ and $\gamma(\frac{1}{\eta_m}\mathbf{y}^m, \mathbf{h}) \geq \gamma(\mathbf{y}^m, \mathbf{h})$, for any $\mathbf{h} \in \mathbb{C}^{2R}$ and $m \in \{1, \ldots, M\}$. Therefore, $\Pr(E|\mathcal{Q}^*, \mathcal{C}^*) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}^{\text{in}*})$, as $\mathcal{C}^*$ is optimal for the region $\mathcal{X}$.

Similary, using $\mathcal{C}^*$, we can construct $\mathcal{C}^{\text{out}} = \{\frac{R}{\|\mathbf{x}^m\|}\mathbf{x}^m : \mathbf{x}^m \in \mathcal{C}^*, m = 1, \ldots, M\}$. Since $\|\mathbf{x}^m\| \leq R$ as $\mathbf{x}^m \in \mathcal{C}^* \subset \mathcal{X}$, $\gamma(\frac{R}{\|\mathbf{x}^m\|}\mathbf{x}^m, \mathbf{h}) \geq \gamma(\mathbf{x}^m, \mathbf{h})$ for any $\mathbf{h} \in \mathbb{C}^{2R}$ and $m \in \{1, \ldots, M\}$. Therefore, $\Pr(E|\mathcal{Q}^*, \mathcal{C}^{\text{out}*}) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}^{\text{out}}) \leq \Pr(E|\mathcal{Q}^*, \mathcal{C}^*)$, as $\mathcal{C}^{\text{out}*}$ is optimal for $\mathcal{X}^{\text{out}}$. ∎

Therefore, the achievable performance with the feasible set $\mathcal{X}$ is bounded by the achievable performance with the feasible sets $\mathcal{X}^{\text{in}}$ and $\mathcal{X}^{\text{out}}$, for any number of feedback bits.

B. BER Analysis for Sum-Power Constrained Relays with Quantized Feedback

We first consider the BER with $\mathcal{C}^{\text{in}} \subset \mathcal{X}^{\text{in}}$. For simplicity, we assume $P_1 = \ldots = P_R = P$. For a particular channel state $\mathbf{h}$, let $\mathbf{y}^m = \mathcal{Q}^*(\mathbf{h}) \in \mathcal{C}^{\text{in}}$ for some $m \in \{1, \ldots, M\}$. Note that, $\mathbf{h} \in \mathcal{R}_m$ by the optimality conditions stated in (3) and (5). In other words, the quantization of the beamforming vector also implies a partition of the channel state space $\mathbb{C}^{2R}$ into M quantization cells. We consider the distribution of the received SNR given $\mathbf{h} \in \mathcal{R}_m$.

Let $b_i = \frac{f_i g_i \sqrt{P}}{\sqrt{1 + |f_i|^2 P_0}}$ and $\mathbf{b} = (b_1, \ldots, b_R)$. We re-write the received SNR given by (1) as:

$$\gamma(\mathbf{y}^m, \mathbf{h}) = \frac{P_0 |\langle \mathbf{b}, \mathbf{y}^m \rangle|^2}{1 + \sum_{i=1}^{R} \frac{|y_i^m|^2 |g_i|^2 P}{1 + |f_i|^2 P_0}} = \frac{P_0 \|\mathbf{b}\|^2 |\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2}{1 + \sum_{i=1}^{R} \frac{|y_i^m|^2 |g_i|^2 P}{1 + |f_i|^2 P_0}} = P_0 |\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2 \sum_{i=1}^{R} \frac{\frac{|g_i|^2 P}{1 + |f_i|^2 P_0}}{1 + \sum_{j=1}^{R} \frac{|y_j^m|^2 |g_j|^2 P}{1 + |f_j|^2 P_0}} \quad (11)$$

where for any complex vector $\mathbf{z}$, we define $\overrightarrow{\mathbf{z}} = \mathbf{z}/\|\mathbf{z}\|$. The second inequality follows from the fact that $\|\mathbf{y}^m\|^2 = 1$. To calculate the average BER, we need the density function of the received SNR. However, the signal and the noise powers of all relays are coupled in the received SNR formula given by (11), which makes the density function calculation intractable, if not possible. For analytical tractability, we seek a decoupled approximation of (11):

$$\gamma(\mathbf{y}^m, \mathbf{h}) \approx P_0 |\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2 \sum_{i=1}^{R} \frac{\frac{|g_i|^2 P}{1 + |f_i|^2 P_0}}{1 + \frac{|g_i|^2 P}{1 + |f_i|^2 P_0} \underbrace{\sum_{j=1}^{R} |y_j^m|^2}_{=1}} = |\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2 \Upsilon^{\text{in}}(\mathbf{h}), \quad (12)$$

where $\Upsilon^{\text{in}}(\mathbf{h}) \triangleq \sum_{i=1}^{R} \frac{|g_i|^2 P_0 P}{1 + |f_i|^2 P_0 + |g_i|^2 P}$. We can see that for the ith term in (11), we approximate the denominator $1 + \sum_{j=1}^{R} \frac{|y_j^m|^2 |g_j|^2 P}{1 + |f_j|^2 P_0}$ by $1 + \frac{|g_i|^2 P}{1 + |f_i|^2 P_0} \sum_{j=1}^{R} |y_j^m|^2 = 1 + \frac{|g_i|^2 P}{1 + |f_i|^2 P_0}$. In other words, for the ith term, we approximate the effect of noises from all relays by the noise from the ith relay only, while preserving the noise power. Notice that the average value of the two are the same. Thus, we have an unbiased approximation for the noise effect. In fact, $\Upsilon^{\text{in}}(\mathbf{h})$ is the maximal received SNR for any channel state $\mathbf{h}$ [11, Eq. 9], i.e., $\Upsilon^{\text{in}}(\mathbf{h}) = \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{X}^{\text{in}})$.

As $\Upsilon^{\text{in}}(\mathbf{h})$ is independent of the quantization cell $\mathcal{R}_m$, *the inner product* $|\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2$ *characterizes the boundaries of the partitions and the performance loss due to quantization.* Therefore, it is sufficient to consider the density of $|\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2$ given $\mathbf{h} \in \mathcal{R}_m$.

For high SNR, the components of $\mathbf{b}$ can be approximated as $b_i = \frac{f_i g_i \sqrt{P}}{\sqrt{1 + |f_i|^2 P_0}} \approx b_i' \triangleq \frac{f_i g_i \sqrt{P}}{\sqrt{|f_i|^2 P_0}} = |g_i| \exp(j(\arg f_i + \arg g_i)) \sqrt{\frac{P}{P_0}}$. Note that this approximation is valid when $|f_i|^2 \gg \frac{1}{P_0}$. For any

finite P_0, since $|f_i|^2$ has an exponential distribution, there is always a non-zero probability that $|f_i|^2 \gg \frac{1}{P_0}$ is not true. However as P_0 increases to infinity, this probability decreases to zero. Thus, we can say that this approximation is valid *almost surely* for high P_0. Hence, if we define $\mathbf{b}' = (b'_1, \ldots, b'_R)$, then we have $|\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2 \approx |\langle \overrightarrow{\mathbf{b}'}, \overrightarrow{\mathbf{y}^m} \rangle|^2$. Together with (12), the actual quantization cells $\mathcal{R}_m$ are approximated as

$$\mathcal{R}_m \approx \mathcal{R}'_m = \left\{ \mathbf{h} : |\langle \overrightarrow{\mathbf{b}'}, \overrightarrow{\mathbf{y}^m} \rangle|^2 > |\langle \overrightarrow{\mathbf{b}'}, \overrightarrow{\mathbf{y}^n} \rangle|^2, \forall n \neq m \right\}.$$

In [26], the conditional density of $\Omega' \triangleq |\langle \overrightarrow{\mathbf{b}'}, \overrightarrow{\mathbf{y}^m} \rangle|^2$ given $\mathbf{h} \in \mathcal{R}'_m$ has been considered for beamforming in MISO systems with quantized feedback. It has been shown that the random variable Ω' has a beta distribution with parameters 1 and $R-1$, and for large B, the conditional PDF of Ω' given $\mathbf{h} \in \mathcal{R}'_m$ can be approximated by

$$f_{\Omega'}(\omega | \mathbf{h} \in \mathcal{R}'_m) = \begin{cases} 2^B (R-1)(1-\omega)^{R-2}, & 1 - 2^{\frac{-B}{R-1}} \leq \omega < 1, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Following the same arguments, we approximate the conditional PDF of $\Omega \triangleq |\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m} \rangle|^2$ given $\mathbf{h} \in \mathcal{R}_m$ by (13), for large P_0 and B as well.

Now let us work on Υ^{in}, which is actually the sum of R independent random variables. The density function for Υ^{in} is simply the convolution of the density functions that we have derived for the proof of Theorem 1 (See (18) in Appendix I). Furthermore, as the norm and the phase of the components of b_i are independent, $\overrightarrow{b}$ is independent of Υ^{in}. Thus, Υ^{in} and Ω are independent. Hence, we can combine (12) and (13) to find the approximated BER as:

$$\begin{aligned} E^{\text{in}} &\triangleq \mathrm{E}_{\Omega, \Upsilon^{\text{in}}} \left[Q \left[\sqrt{2\beta\Omega\Upsilon^{\text{in}}} \right] \right] = \mathrm{E}_\Omega \left[\mathrm{E}_{\Upsilon^{\text{in}}} \left[Q \left[\sqrt{2\beta\Omega\Upsilon^{\text{in}}} \right] \Big| \Omega = \omega \right] \right] \\ &= 2^B (R-1) \int_{1-2^{\frac{-B}{R-1}}}^1 (1-\omega)^{R-2} \left[\int_0^\infty Q\left[2\beta\omega\upsilon \right] f_{\Upsilon^{\text{in}}}(\upsilon) d\upsilon \right] f_\Omega(\omega) d\omega, \end{aligned} \tag{14}$$

which can easily be calculated numerically.

A similar analysis can be carried out for $\Pr(E; \mathcal{Q}^*, \mathcal{C}^{\text{out}})$. Then, it is easy to show that

$$E^{\text{out}} \triangleq \mathrm{E}_{\Omega, \Upsilon^{\text{out}}} \left[Q \left[\sqrt{2\beta\Omega\Upsilon^{\text{out}}} \right] \right], \quad \text{where } \Upsilon^{\text{out}} \triangleq \sum_{i=1}^R \frac{|g_i|^2 R P_0 P}{1 + |f_i|^2 P_0 + |g_i|^2 R P} \tag{15}$$

According to the heuristic discussions on the validity of these approximations, we expect them to work well for large B and high SNR. Actually, the simulations in Section-VII show that they are even valid for moderate B and low SNR.

It is also worth to point out that, actually (14) is an accurate approximation on the BER for networks with a sum-power constraint on the relays. In Section-VII, we also show the simulated BER of networks with a sum-power constraint and compare this with the approximated BER.

VII. SIMULATION RESULTS

In this section, numerical results are provided. Unless otherwise specified, we assume an equal power constraint on both the transmitter and the relays. In other words, $P_0 = P_1 = \cdots = P_R = P$. The horizontal axis of the figures indicates P, and the vertical axis represents the average BER.

Fig. 3 shows the simulation results for networks with individual power constraints on relays. The GLA is used to design the quantizer codebook for the feasible set $\mathcal{X}$. To avoid bad suboptimal codebook designs, we also use simulated annealing [30]. We can see that the relay selection achieves full diversity order in all cases. However, with the same number of feedback bits, the relay selection scheme is suboptimal compared to network beamforming using the GLA. This can be observed for networks with 2 and 4 relays, and 1 and 2 feedback bits, respectively. Also notice that a few feedback bits can result in a very close performance to the optimal case (∞ bits). At a BER of 10^{-5}, the network with 3 relays and 4 bits/8 bits feedback bits is only 1.5dB/0.5dB worse than the optimal.

The GLA can also be used to design codebooks for networks with unequal individual power constraints. In Fig. 4, we show the performance results for a 2-relay network. The power constraints of the source and the relays are chosen as $P_0 = P_1 = P$ and $P_2 = \frac{P}{2}$. In Fig. 5, we show the performance of a 3-relay network in which $P_0 = P$ and $P_1 = P_2 = P_3 = \frac{P}{3}$. In either case, both the relay selection scheme and the feedback schemes achieve full diversity.

In Fig. 6, we show the performance of networks with a sum-power constraint on relays. The reasons for this are two-fold. First, we show that the GLA can also be used to design codebooks for networks with a sum-power constraint on relays, and second, we show the validity of our BER approximations in (14) and (15). First, we can see that network beamforming using the

GLA works for this network model as well. Full diversity is achieved for 2, 4, and 8 bits of feedback in 3-relay networks. Compared to the infinite feedback bits case, network beamforming with 8 feedback bits is less than 0.5 dB worse. In Fig. 6, we also compare the simulated BER of the GLA designs with (14), for networks with 3 relays. We can observe that the approximation provides a lower bound on the BER in all of the cases. This is intuitively clear, as similar to [26], the random variable Ω overestimates the distribution of $|\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m}\rangle|^2$ within each quantization cell. Moreover, $\Upsilon^{\mathrm{in}}(\mathbf{h})$ is the maximal received SNR for any channel state $\mathbf{h}$, and as $B \to \infty$, we have $|\langle \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{y}^m}\rangle|^2 \to 1$, and the analysis remains consistent with the actual performance. The approximated BERs are accurate even when B is small, within 1 dB for all of the cases.

Finally, we compare the bounds in (14) and (15) with the simulated BER for networks with individual power constraints in Fig. 7. In both cases, the upper bound and the lower bound is within 1 dB and 1.5 dB of the actual BER, respectively. Notice that the performance of the lower bound is better than the performance with ∞ bits feedback. The reason is that the lower bound corresponds to the feasible set $\mathcal{X}^{\mathrm{out}}$, which can provide a higher array gain compared to the actual feasible set $\mathcal{X}$. On the other hand, as the performance with ∞ bits is not known analytically, the upper and lower bound gives valuable information about the performance of our limited feedback scheme. We also note that, as a result of the nature of our approximations, the upper bound may not work well when B is small.

VIII. Conclusion

We study quantized beamforming schemes for wireless amplify-and-forward relay networks. The Generalized Lloyd Algorithm is used for the codebook design procedure. We rigorously analyzed the achievable performance in terms of the diversity order, the average SNR loss and the capacity loss. We also provided approximate upper and lower bounds on the bit error rate, which can be calculated numerically. Our analytical results were verified by simulations. It is observed that our design methods achieve full diversity order and a few feedback bits is sufficient to achieve a satisfactory performance in terms of the array gain. We also observed that our approximate BER analysis provides an accurate characterization of the performance, even for moderate values of B.

APPENDIX I

PROOF OF THEOREM 1

For simplicity of notations, define the random variables $X_i \triangleq \zeta_i^{-1}|f_i|^2$ and $Y_i \triangleq \xi_i^{-1}|g_i|^2$, $\zeta_i, \xi_i > 0$, $\forall i$, which implies $X_i \sim \text{Exp}(\zeta_i)$ and $Y_i \sim \text{Exp}(\xi_i)$. Note that (8) will follow from the substitutions $\zeta_i = 1$ and $\xi_i = \lambda_i^{-1}$. Now, the relay selection function can be expressed as $H_i \triangleq \frac{\gamma^2 X_i Y_i}{1 + \gamma X_i + \gamma Y_i}$, $i = 1, \ldots, R$, where $\gamma \triangleq P_0$. Thus, H_i is a random variable which is also the received SNR given that the ith relay is selected.

Lemma 4: The cumulative distribution function of H_i is given by:

$$\Pr(H_i \leq \hat{\gamma}) = \begin{cases} 1 - \exp\left(-\frac{\hat{\gamma}}{\gamma}(\zeta_i + \xi_i)\right) \kappa K_1(\kappa), & \hat{\gamma} \geq 0, \\ 0, & \hat{\gamma} < 0. \end{cases} \quad \text{where } \kappa = \sqrt{\frac{4\zeta_i \xi_i \hat{\gamma}(1+\hat{\gamma})}{\gamma^2}}. \quad (16)$$

Proof: Let $X \sim \text{Exp}(\zeta)$ and $Y \sim \text{Exp}(\xi)$. Define $\bar{H} \triangleq \frac{\gamma^2 XY}{1+\gamma X+\gamma Y}$, where γ is a constant. Then, $\Pr(\bar{H} \leq \hat{\gamma}) = \Pr\left(\frac{\gamma^2 XY}{1+\gamma X+\gamma Y} \leq \hat{\gamma}\right) = \int_0^\infty \Pr(\gamma^2 xY - \gamma\hat{\gamma}Y \leq \hat{\gamma} + \gamma\hat{\gamma}x | X = x) f_X(x) dx$. Note that $\Pr(\gamma^2 xY - \gamma\hat{\gamma}Y \leq \hat{\gamma} + \gamma\hat{\gamma}x | X = x, X < \hat{\gamma}/\gamma) = 1$. Therefore,

$$\Pr(\bar{H} \leq \hat{\gamma}) = \int_0^{\hat{\gamma}/\gamma} f_X(x) dx + \int_{\hat{\gamma}/\gamma}^\infty \Pr\left(Y \leq \frac{\hat{\gamma} + \gamma\hat{\gamma}x}{\gamma^2 x - \gamma\hat{\gamma}} | X = x\right) f_X(x) dx. \quad (17)$$

The first integral is obvious since $f_X(x) = \zeta e^{-\zeta x}$. For the second integral, we substitute $u = x - \hat{\gamma}/\gamma$ and use $\Pr(Y \leq y) = 1 - e^{-\xi y}$ for $y \geq 0$. Thus,

$$\Pr(\bar{H} \leq \hat{\gamma}) = 1 - e^{-\zeta\frac{\hat{\gamma}}{\gamma}} + \zeta e^{-\zeta\frac{\hat{\gamma}}{\gamma}} \int_0^\infty \left[1 - \exp\left(-\frac{\xi}{u}\frac{\hat{\gamma}(1+\hat{\gamma})}{\gamma^2} - \frac{\xi\hat{\gamma}}{\gamma}\right)\right] e^{-\zeta u} du,$$

which can be evaluated using $\int_0^\infty \exp\left(-\frac{\alpha}{4x} - \beta x\right) dx = \sqrt{\frac{\alpha}{\beta}} K_1\left(\sqrt{\alpha\beta}\right)$, for $\alpha, \beta > 0$ [31, Eq. 3.324.1]. Then, the result follows after some simplifications. The discontinuity of the function $K_1(\kappa)$ at $\kappa = 0$ for $\hat{\gamma} = 0$ can be avoided by a direct evaluation of (17). ∎

It is now easy to calculate the PDF of H_i using $K_1'(x) = -K_0(x) - x^{-1} K_1(x)$ [32, Eq. 9.6.26] and differentiating (16) with respect to $\hat{\gamma}$.

$$f_{H_i}(\hat{\gamma}) = \exp\left(\frac{-\hat{\gamma}}{\gamma}(\zeta_i + \xi_i)\right) \gamma^{-2} \left[\kappa\gamma(\zeta_i + \xi_i) K_1(\kappa) + 2\zeta_i \xi_i K_0(\kappa)(1 + 2\hat{\gamma})\right]. \quad (18)$$

The received SNR for the relay selection scheme is given by the random variable $H \triangleq \max_i H_i(\gamma)$. Since $H_i(\gamma)$ are independent,

$$f_H(\hat{\gamma}) = \sum_{i=1}^R \left[\prod_{j=1, j\neq i}^R \Pr(H_j \leq \hat{\gamma})\right] f_{H_i}(\hat{\gamma}) \quad (19)$$

by the rule for the derivative of a product. Now, the BER $\Pr(E)$ can be expressed as

$$\Pr(E) = \mathrm{E}_{\hat{\gamma}}\left[\mathrm{Q}\left(\sqrt{2\beta H}\right)\right] = \int_0^\infty f_H(\hat{\gamma})\mathrm{Q}\left(\sqrt{2\beta\hat{\gamma}}\right)d\hat{\gamma}. \tag{20}$$

We want to find an upper bound for (20). As an intermediate step, we have:

Lemma 5: The following bounds apply for $K_\nu(x)$, $\nu \in \mathbb{Z}^+$, $\forall x > 0$, $x \in \mathbb{R}$:

$$K_\nu(x) \le 2^{\nu-1}\Gamma(\nu)x^{-\nu}, \tag{21}$$

$$xK_1(x) \ge e^{-x}, \tag{22}$$

$$K_0(x) \le 2x^{-1}. \tag{23}$$

Proof: We start with (21), and use an integral equality for the function $K_\nu(x)$ [32, Eq. 9.6.25]: $K_\nu(x) = \frac{2^\nu\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}x^\nu}\int_0^\infty \cos(xt)(t^2+1)^{-\nu-\frac{1}{2}}dt \le \frac{2^\nu\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}x^\nu}\int_0^\infty (t^2+1)^{-\nu-\frac{1}{2}}dt = 2^{\nu-1}\Gamma(\nu)x^{-\nu}$, where [31, Eq. 3.241] is used to evaluate the last integral.

For (22), we use $K_1(x) = x\int_1^\infty e^{-xt}\sqrt{t^2-1}dt$ [32, Eq. 9.6.23]. As $\sqrt{t^2-1} \ge t-1$ for $t \ge 1$, $K_1(x) \ge x\int_1^\infty e^{-xt}(t-1)\,dt = x\frac{e^{-x}}{x^2} = \frac{e^{-x}}{x}$.

For (23), we use the following recurrence relation [32, Eq. 9.6.26]: $\mathcal{L}_{\nu-1}(x) - \mathcal{L}_{\nu+1}(x) = \frac{2\nu}{x}\mathcal{L}_\nu(x)$, $\mathcal{L}_v \triangleq e^{i\pi\nu}K_\nu$. Substituting $\nu = 1$, $K_0(x) = K_2(x) - 2x^{-1}K_1(x) \le 2x^{-2} - 2x^{-2}e^{-x} \le 2x^{-1}$, where for the first inequality we have used (21) and (22) to bound $K_2(x)$ and $-K_1(x)$, respectively. The last inequality follows from $1 - e^{-x} \le x$ for $x \ge 0$. ■

Now, we bound $f_{H_i}(\hat{\gamma})$ in (18) as

$$f_{H_i}(\hat{\gamma}) \le \frac{1}{\gamma}\left[\zeta_i + \xi_i + 2\sqrt{\zeta_i\xi_i}\frac{1+2\hat{\gamma}}{\sqrt{\hat{\gamma}(1+\hat{\gamma})}}\right], \tag{24}$$

where we used $\exp\left(\frac{-\hat{\gamma}}{\gamma}(\zeta_i + \xi_i)\right) \le 1$, and (21) and (23) to bound $\kappa K_1(\kappa)$ and $K_0(\kappa)$. Similary, for $\Pr(H_i \le \hat{\gamma})$ given in Lemma 4, we have

$$\Pr(H_i \le \hat{\gamma}) \le \frac{1}{\gamma}\left[\hat{\gamma}(\zeta_i + \xi_i) + 2\sqrt{\zeta_i\xi_i}\sqrt{\hat{\gamma}(1+\hat{\gamma})}\right]. \tag{25}$$

Substituting (24) and (25) to (19), we have the representation $f_H(\hat{\gamma}) \le \Psi(\hat{\gamma})\gamma^{-R}$, where $\Psi(\hat{\gamma})$ is independent of γ. Next, we show that $\Psi(\hat{\gamma})$ is bounded by a polynomial of degree $(R-1)$ of $\hat{\gamma}$ for $R \ge 2$. Let $a = \max_i\{\zeta_i + \xi_i\}$, $b = \max_i\left\{2\sqrt{\zeta_i\xi_i}\right\}$. Then, we have

$$\Psi(\hat{\gamma}) \le R\left[a\hat{\gamma} + b\sqrt{\hat{\gamma}(1+\hat{\gamma})}\right]^{R-1}\left[a + b\frac{1+2\hat{\gamma}}{\sqrt{\hat{\gamma}(1+\hat{\gamma})}}\right]$$

$$\leq R \left[a\sqrt{\hat{\gamma}(1+\hat{\gamma})} + b\sqrt{\hat{\gamma}(1+\hat{\gamma})} \right]^{R-1} \left[a + b\frac{1+2\hat{\gamma}}{\hat{\gamma}} \right]$$

$$= R \left(a+b \right)^{R-1} \hat{\gamma}^{\frac{R-3}{2}} \left(1+\hat{\gamma} \right)^{\frac{R-1}{2}} \left(\hat{\gamma}(a+2b) + b \right)$$

$$\leq 2R \left(a+b \right)^{R} \hat{\gamma}^{\frac{R-3}{2}} \left(1+\hat{\gamma} \right)^{\frac{R+1}{2}} \leq 2R \left(a+b \right)^{R} \left(1+\hat{\gamma} \right)^{R-1}. \tag{26}$$

Now let $\lambda = \max_i \left\{ \sqrt{\zeta_i} + \sqrt{\xi_i} \right\} = (a+b)^{\frac{1}{2}}$. Combining (20) and (26),

$$\Pr(E) \leq \gamma^{-R} \int_0^\infty \Psi\left(\hat{\gamma}\right) Q\left(\sqrt{2\beta\hat{\gamma}}\right) d\hat{\gamma}$$

$$\leq \gamma^{-R} R\lambda^{2R} \int_0^\infty \left(1+\hat{\gamma}\right)^{R-1} e^{-\beta\hat{\gamma}} d\hat{\gamma}$$

$$= \gamma^{-R} R\lambda^{2R} \sum_{n=0}^{R-1} \binom{R-1}{n} \int_0^\infty \hat{\gamma}^{R-1-n} e^{-\beta\hat{\gamma}} d\hat{\gamma}$$

$$= \gamma^{-R} R\lambda^{2R} \sum_{n=0}^{R-1} \binom{R-1}{n} \Gamma(R-n)\beta^{-(R-n)} = \gamma^{-R} R\lambda^{2R} \sum_{n=0}^{R-1} \binom{R-1}{n} \beta^{-(n+1)} n!,$$

which concludes the proof.

APPENDIX II

PROOF OF THEOREM 2

First, we will introduce a suboptimal quantizer $\mathcal{Q}_u : \mathbb{C}^{2R} \rightarrow \mathcal{C}_u$ to simplify the proof. We similarly define the pointwise SNR loss for $\mathcal{C}_u$ with the quantizer $\mathcal{Q}_u$ as $D_u(\mathbf{h}) \triangleq \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{X}) - \gamma(\mathbf{h}; \mathcal{Q}_u, \mathcal{C}_u)$. Now, since $\gamma(\mathbf{h}; \mathcal{Q}_u, \mathcal{C}_u) \leq \gamma(\mathbf{h}; \mathcal{Q}^*, \mathcal{C}_u)$, we also have $D(\mathbf{h}) \leq D_u(\mathbf{h})$, $\forall \mathbf{h} \in \mathbb{C}^{2R}$, which implies $E_{\mathbf{h}}\left[D(\mathbf{h})\right] \leq E_{\mathbf{h}}\left[D_u(\mathbf{h})\right]$. Therefore, it suffices to consider the average SNR loss for the system with codebook $\mathcal{C}_u$ and the quantizer $\mathcal{Q}_u$.

Let $\mathbf{y}$ denote the optimal beamforming vector for some channel state $\mathbf{h}$. By definition, we have $|y_q| = 1$ for some $q \in \{1, \ldots, R\}$. Let $\arg y_1 = 0$, without loss of generality.

Let $w \in \mathbb{R}$, and consider $\exp(jw)$ as a point on the complex unit circle. We define $\mathcal{V}(w)$ as the nearest neighbor of $\exp(jw)$ in the set $\{\exp(jv)\}$, $v \in V$, i.e. $\mathcal{V}(w) = \arg\max_{v \in V} \cos(w - v)$. Then, our suboptimal quantized beamforming vector $\mathbf{p} \triangleq \mathcal{Q}_u(\mathbf{h})$ is defined by:

$$|p_i| = \max\left\{ u : u \leq |y_i| \text{ and } u \in U \right\}, \arg p_i = \begin{cases} 0, & i = 1, \\ \mathcal{V}\left(\arg y_i + \theta_q\right), & i = 2, \ldots, R. \end{cases}$$

where $\theta_q = \mathcal{V}(\arg y_q) - \arg y_q$. Note that $\mathcal{Q}_u$ guarantees $\mathbf{p} \in \mathcal{C}_u$ and the components of $\mathbf{p}$ satisfy $|p_q| = |y_q| = 1$, and $|y_i| - |p_i| \leq K^{-1}$ for $i \neq q$.

We first show a property of our quantization scheme. Define $\mathbf{x} = \exp(-j\theta_q)\mathbf{p}$, and note that $\gamma(\mathbf{x}, \mathbf{h}) = \gamma(\mathbf{p}, \mathbf{h})$, even though $\mathbf{x} \notin \mathcal{C}_u$. Therefore, $D_u(\mathbf{h}) = \gamma(\mathbf{y}, \mathbf{h}) - \gamma(\mathbf{p}, \mathbf{h}) = \gamma(\mathbf{y}, \mathbf{h}) - \gamma(\mathbf{x}, \mathbf{h})$. Thus, for simplicity, we will use $\mathbf{x}$ for analysis, instead of $\mathbf{p}$. Similarly, we have $|x_q| = |y_q| = 1$, $|y_i| - |x_i| \le K^{-1}$ for $i \ne q$. However, $\arg x_q = \arg p_q - \theta_q = \arg p_q - (\mathcal{V}(\arg y_q) - \arg y_q) = \arg y_q$ as $\arg p_q = \mathcal{V}(\arg y_q + \theta_q) = \mathcal{V}(\arg y_q + \mathcal{V}(\arg y_q) - \arg y_q) = \mathcal{V}(\mathcal{V}(\arg y_q)) = \mathcal{V}(\arg y_q)$. Therefore, $x_q = y_q$. Also, $\arg y_i - \arg x_i = \arg y_i - \arg p_i + \theta_q = \arg y_i + \theta_q - \mathcal{V}(\arg y_i + \theta_q)$, which implies $|\arg y_i - \arg x_i| \le \pi L^{-1}$ for $i \ne q$, by the definition of $\mathcal{V}$.

For the next step, we decompose $D_u(\mathbf{h}) = \gamma(\mathbf{y}, \mathbf{h}) - \gamma(\mathbf{x}, \mathbf{h})$ as:

$$D_u(\mathbf{h}) = P_0 \sum_{i=1}^{R} D_u^{(i)}(\mathbf{h}), \text{ where } D_u^{(i)}(\mathbf{h}) \triangleq \gamma(\mathbf{w}^{(i-1)}, \mathbf{h}) - \gamma(\mathbf{w}^{(i)}, \mathbf{h}), \tag{27}$$

and $\mathbf{w}^{(0)} = \mathbf{y}$, $\mathbf{w}^{(i)} = (x_1, \ldots, x_i, y_{i+1}, \ldots, y_R)$ for $i = 1, \ldots, R-1$, and $\mathbf{w}^{(R)} = \mathbf{x}$. In other words, we successively alter each component of $\mathbf{y}$, until we reach $\mathbf{x}$. First, we will find an upper bound for $D_u^{(j)}(\mathbf{h})$, $j \in \{1, \ldots, R\}$, $j \ne q$. Note that $D_u^{(q)}(\mathbf{h}) = 0$ as $x_q = y_q$.

Let $\delta_K \triangleq |y_j| - |x_j| \le K^{-1}$ and $\delta_L \triangleq |\arg y_j - \arg x_j| \le \pi L^{-1}$. Also, $\rho_i \triangleq \frac{P_i}{1 + |f_i|^2 P_0}$ and

$A \triangleq \sum_{i=1}^{R} w_i^{(j-1)} f_i g_i \sqrt{\rho_i}$ $\qquad A' \triangleq \sum_{i=1}^{R} w_i^{(j)} f_i g_i \sqrt{\rho_i} = A - (y_j - x_j) f_j g_j \sqrt{\rho_j}$

$B \triangleq 1 + \sum_{i=1}^{R} |w_i^{(j-1)}|^2 |g_i|^2 \rho_i$ $\quad B' \triangleq 1 + \sum_{i=1}^{R} |w_i^{(j)}|^2 |g_i|^2 \rho_i = B - (|y_j|^2 - |x_j|^2)|g_j|^2 \rho_j,$

so that $D_u^{(j)}(\mathbf{h}) = \frac{|A|^2}{B} - \frac{|A'|^2}{B'}$ can be bounded as

$$D_u^{(j)}(\mathbf{h}) = -\frac{|A|^2 \left(|y_j|^2 - |x_j|^2\right) |g_j|^2 \rho_j}{BB'} + \frac{2\mathfrak{Re}\left\{A^*(y_j - x_j) f_j g_j \sqrt{\rho_j}\right\}}{B'} - \frac{|y_j - x_j|^2 |f_j|^2 |g_j|^2 \rho_j}{B'}$$

$$\le \frac{2\mathfrak{Re}\left\{A^*(y_j - x_j) f_j g_j \sqrt{\rho_j}\right\}}{B'} \le 2\mathfrak{Re}\left\{A^*(y_j - x_j) f_j g_j \sqrt{\rho_j}\right\} \le 2|A||y_j - x_j||f_j||g_j|\sqrt{\rho_j}. \tag{28}$$

Define $\lambda_i = \sqrt{\frac{P_i}{P_0}}$. Then, we use the following inequalities to further bound (28):

$$|f_i|\sqrt{\rho_i} = |f_i|\sqrt{\frac{P_i}{1 + |f_i|^2 P_0}} \le \sqrt{\lambda_i}, \text{ so that } |A| \le \sum_{i=1}^{R} |w_i^{(j-1)}||f_i||g_i|\sqrt{\rho_i} \le \sum_{i=1}^{R} |g_i|\sqrt{\lambda_i}, \tag{29}$$

$$|y_j - x_j| = \sqrt{\delta_K^2 + 2|y_j||x_j|(1 - \cos\delta_L)} \le \sqrt{\delta_K^2 + |y_j||x_j|\delta_L^2} \le \sqrt{\delta_K^2 + \delta_L^2} \le \sqrt{\frac{1}{K^2} + \frac{\pi^2}{L^2}}. \tag{30}$$

For (30), the first inequality can easily be verified using the Taylor series expansion for $\cos\delta_L$, and we used $|x_j||y_j| \le 1$ for the second inequality. Let us choose $L = K$. Using (29) and (30) in (28), $D_u^{(j)}(\mathbf{h}) \le 2K^{-1}\sqrt{1 + \pi^2}|g_j|\sqrt{\lambda_j} \sum_{i=1}^{R} |g_i|\sqrt{\lambda_i}$, which is independent of $\mathbf{y}$ and $\mathbf{x}$. This

provides a tractable calculation of the expectation as follows: Define $Z_i \triangleq |g_i|^2$. Then,

$$\mathrm{E}_\mathbf{h}[D_u^{(j)}(\mathbf{h})] = \frac{2\sqrt{1+\pi^2}}{K}\left[\mathrm{E}_{z_j}[Z_j]\,\lambda_j + \mathrm{E}_{z_j}\left[Z_j^{\frac{1}{2}}\right]\lambda_j^{\frac{1}{2}}\sum_{i=1,\,i\neq j}^{R}\mathrm{E}_{z_i}\left[Z_i^{\frac{1}{2}}\right]\lambda_i^{\frac{1}{2}}\right]$$

$$= \frac{2\sqrt{1+\pi^2}}{K}\left[\Gamma\left(2\right)\lambda_j + \Gamma\left(\tfrac{3}{2}\right)\lambda_j^{\frac{1}{2}}\sum_{i=1,\,i\neq j}^{R}\Gamma\left(\tfrac{3}{2}\right)\lambda_i^{\frac{1}{2}}\right],$$

as $\mathrm{E}_{z_i}\left[Z_i^a\right] = \int_0^\infty z_i^a e^{-z_i}dz_i = \Gamma(a+1)$. Now, let $\lambda' \triangleq \max_i \lambda_i$. For any $j \in \{1, \ldots, R\}$, we have

$$\mathrm{E}_\mathbf{h}\left[D_u^{(j)}(\mathbf{h})\right] \le D_u', \text{ where } D_u' = 2\lambda' K^{-1}\sqrt{1+\pi^2}\left[1 + (R-1)\Gamma^2\left(\tfrac{3}{2}\right)\right],$$

which is independent of j. Then, using (27) and the fact that $D_u^{(q)}(\mathbf{h}) = 0$,

$$\mathrm{E}_\mathbf{h}\left[D(\mathbf{h})\right] \le \mathrm{E}_\mathbf{h}\left[D_u(\mathbf{h})\right] \le P_0(R-1)D_u'. \tag{31}$$

Since we set $L = K$, $M = K^{R-1}\left[K^R - (K-1)^R\right]$. Next, we show that $K^R - (K-1)^R \le RK^{R-1}$ for any $K, R \ge 1$. We prove this by induction. The inequality is obvious for $K = 1$. For $K \ge 2$, assume that it is true for k. Then, $(k+1)^R - k^R = \sum_{n=1}^{R}\binom{R}{n}k^{R-n} \le \sum_{n=1}^{R}n\binom{R}{n}k^{R-n} = R\sum_{n=0}^{R-1}\binom{R-1}{n}k^{R-1-n} = R(k+1)^{R-1}$, which shows that the inequality is true for $k+1$. Thus, we have proved the inequality.

Therefore, $M \le RK^{2(R-1)}$, or equivalently, $K^{-1} \le 2^{-\frac{B}{2(R-1)}}R^{\frac{1}{2(R-1)}}$. We can re-write (31) as $\mathrm{E}_\mathbf{h}[D(\mathbf{h})] \le \mu 2^{-\frac{B}{2(R-1)}}$, where $\mu = 2P_0\sqrt{1+\pi^2}\lambda'(R-1)R^{\frac{1}{2(R-1)}}\left[1 + (R-1)\Gamma^2\left(\tfrac{3}{2}\right)\right]$. This concludes the proof.

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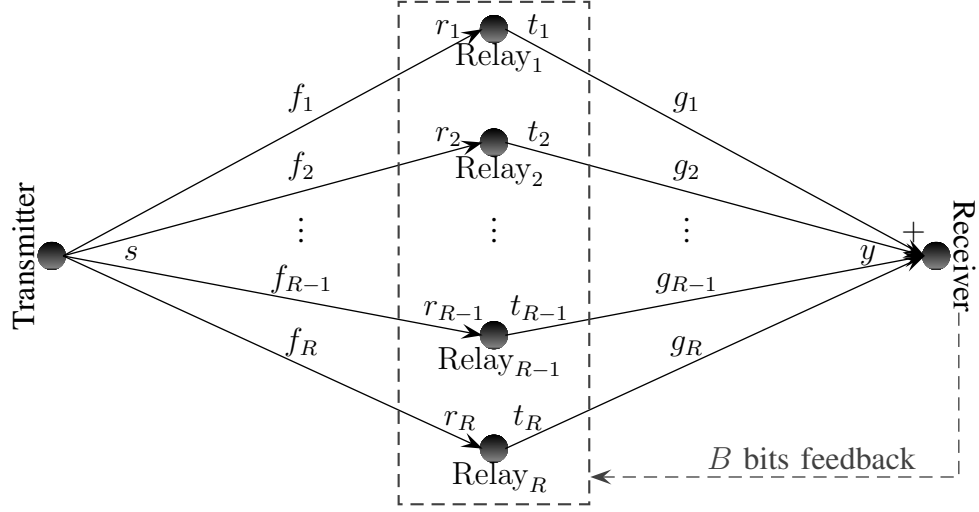[32] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions*. National Bureau of Standards, Applied Mathematics Series-55, 1972.
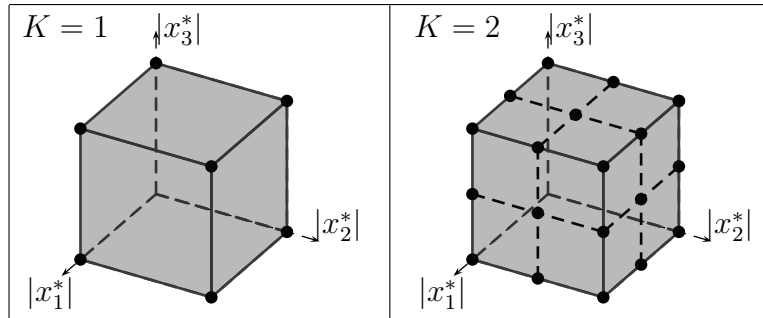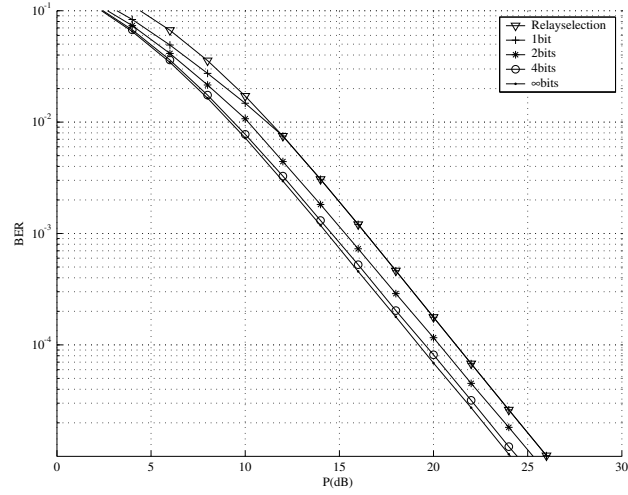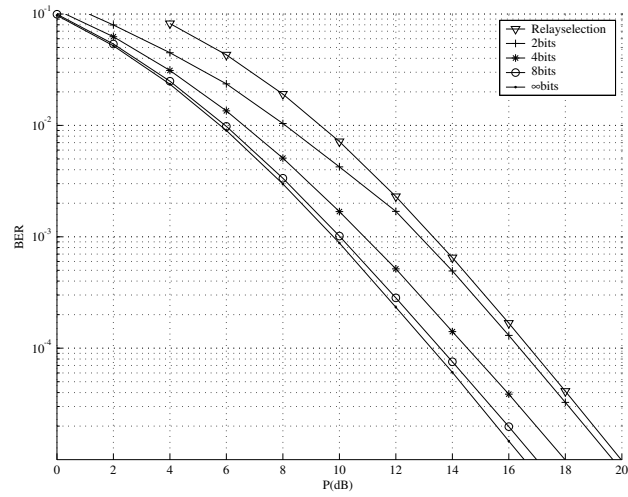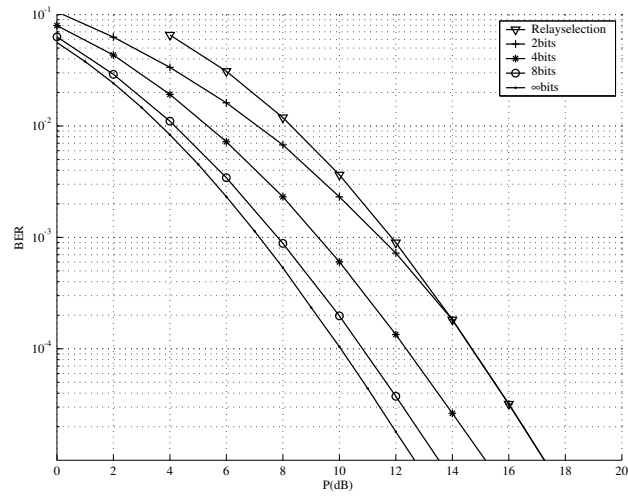
Fig. 1: System block diagram

Fig. 2: Quantized power codebook design for $R = 3$ and $K = 1, 2$. The shaded surface regions represent the set of all feasible power control vectors. The power codebook vectors are expressed by points on this region.

(a) 2-relay network

(b) 3-relay network

(c) 4-relay network

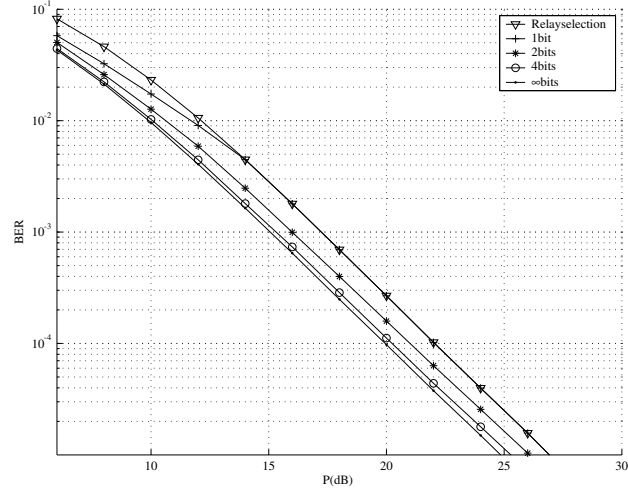Fig. 3: Performance results for networks with individual power constraints

Fig. 4: Performance results for a 2-relay network with unequal individual power constraints: $P_0 = P_1 = P$, $P_2 = \frac{P}{2}$.
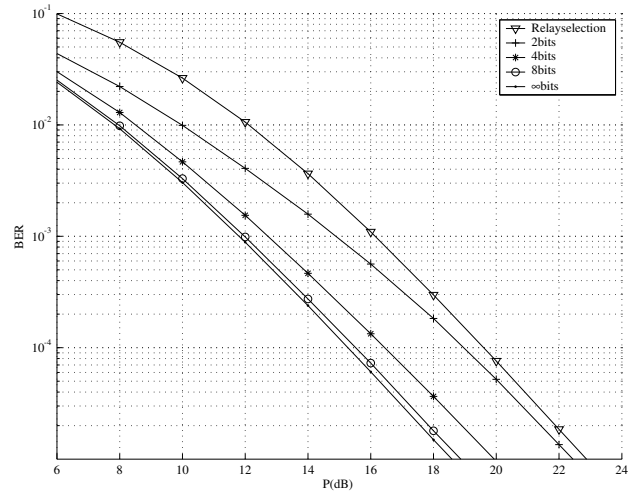
Fig. 5: Performance results for a 3-relay network with unequal individual power constraints: $P_0 = P$, $P_1 = P_2 = P_3 = \frac{P}{3}$.
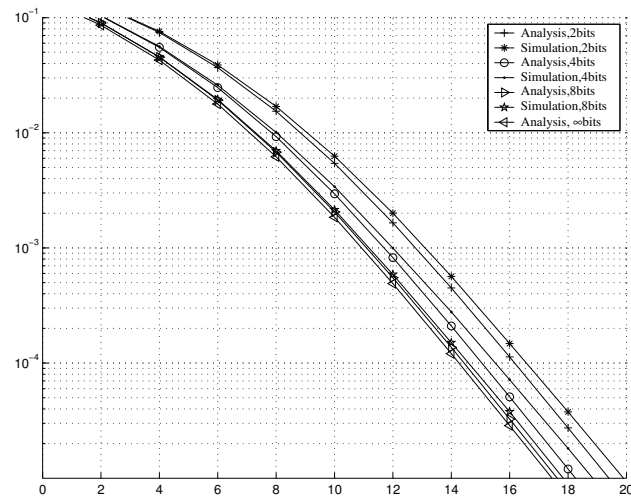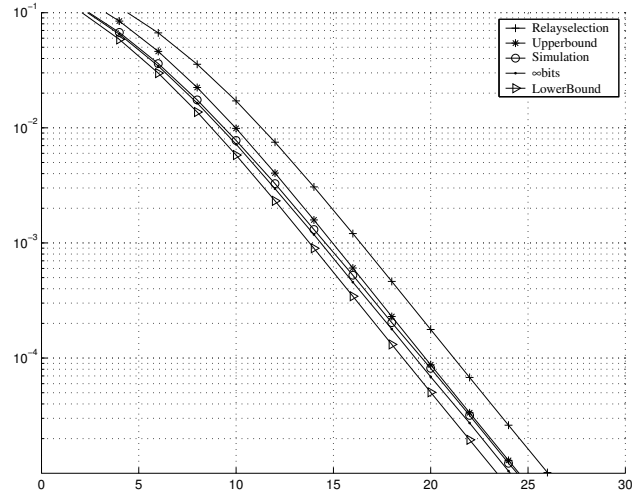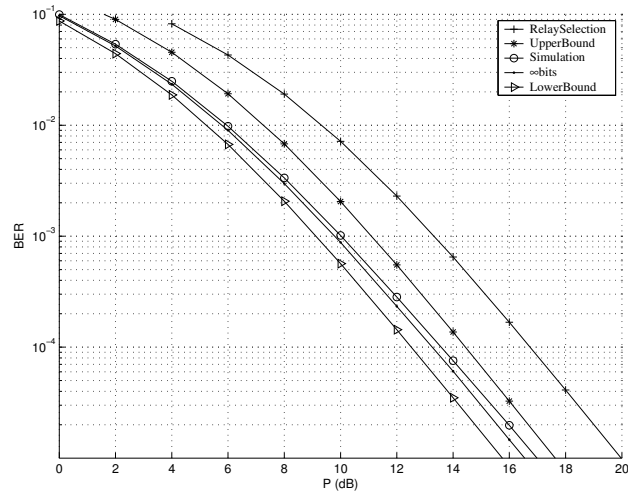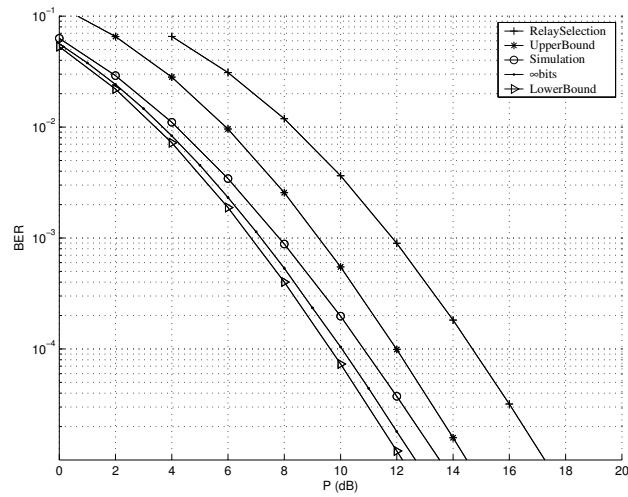
Fig. 6: BER analysis and simulations for a 3-relay network with sum-power constraint

(a) 2-relay networks, 4 feedback bits

(b) 3-relay networks, 8 feedback bits

(c) 4-relay networks, 8 feedback bits

Fig. 7: BER analysis for networks with individual power constraints